

THEORY OF NUMERICAL SETS

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“Numerical sets” is the most interesting theme of mathematical calculus.

Let's read it.

Methods of defining sets. A set can be defined in either of two ways:

a) explicitly listing each of the elements of the set e.g. the set $\{2, 3, 4, 5\}$.

b) describing the set by stating properties that define it e.g. the set of all black cats in France.

The first method is called the *roster method* and the second method is called the *property method*.

Sets are usually denoted by capital letters such as A, B, X, S, etc. and the elements within them by lower case letters such as a, b, x, s, etc. When the roster method is used to define a set, the elements of the set are usually enclosed in braces and separated by commas e.g. $S = \{3, 5, 7, 9\}$ is the set S consisting of elements 3, 5, 7, 9. The notation $x \in S$ means that x is an element of S; $x \notin S$ means x is not an element of S. To indicate a set of objects having the property P, the notation $\{x \mid x \text{ has the property P}\}$ is used. The notation $\{x \mid \}$ is called a *set builder*. The bar “ \mid ” is read “such that.”

Example. $S = \{x \mid x \text{ is an even integer}\}$, which is the set of all even integers. Some authors use the colon : instead of the bar i.e $S = \{x: x \text{ is an even integer}\}$.

Two sets A and B are said to be equal if every element of A is an element of B and every element of B is an element of A. The equality of

sets A and B is denoted by $A = B$. Inequality of two sets A and B is denoted by $A \neq B$.

Examples.

- The sets $A = \{3, 4, 5, 6\}$ and $B = \{5, 3, 4, 6\}$ are equal since the order in which set elements are listed is immaterial.
- The sets $A = \{4, 5, 6\}$ and $B = \{4, 5, 5, 6\}$ are equal since repeating an element of a set does not change the set.

Let S be a given set. Any set A , each of whose elements is also an element of S , is said to be *contained* in S and called a *subset* of S .

Example. The sets $A = \{5\}$, $B = \{3, 4, 5\}$, and $C = \{6, 7\}$ are all subsets of $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Reviewing the definition we note that the entire set S qualifies as a subset of S . Thus any set is a subset of itself.

We write “ A is a subset of S ” as $A \subset S$.

If A is a subset of S and $A \neq S$, then A is called a proper subset of S .

In the above example sets A , B and C are all proper subsets of S .

If A is a subset of S , then we can also write $S \supset A$ which is read “ S is a superset of A ”.

Note. Some authors use $A \subseteq S$ to indicate that A is a subset of S and reserve $A \subset B$ to indicate that A is a proper subset of S .

Theorem. If $A \subset B$ and $B \subset C$, then $A \subset C$.

However illogical it may seem, it is convenient and useful to have the concept of an empty set, a set containing no elements. We call such a set the *empty* (or *null*) set and denote it by \emptyset .

The null set \emptyset is considered to be a subset of every set.

Often a discussion involves subsets of some particular set called the *universe of discourse* (or briefly *universe*), *universal set* or *space*. The elements of a space are often called the *points of the space*. We denote the universal set by U .

Example. The set of all even integers could be considered a subset of a universal set consisting of all the integers. Or they could be considered a

subset of a universal set consisting of all the rational numbers. Or of all the real numbers.

Often the universal set may not be explicitly stated and it may be unclear as to just what it is. At other times it will be clear.

The complement of a set A with respect to a given universal set U is the set of elements in U that are not in A . The complement of A is typically denoted by A^c or A' .

A *finite set* is a set with a finite number of elements and an *infinite set* is one with an infinite number of elements.

Examples.

- The set of all black cats in France is a finite set.
- The set of all even integers is an infinite set.

Two sets A and B are said to be *comparable* if $A \subset B$ or $B \subset A$ i.e. if one of the sets is a subset of the other. Two sets A and B are said to be *not comparable* if $A \not\subset B$ and $B \not\subset A$.

We thus note that if two sets A and B are not comparable there is necessarily an element in A that is not in B and an element in B that is not in A .

Two sets are called *disjoint* if they have no elements in common i.e. the intersection of the sets is the null set. A system of more than two sets is *pairwise disjoint* (sometimes called simply *disjoint*) if every pair of sets in the system is disjoint.

Sometimes the members of a set are sets themselves. If the members of a set A are themselves sets it is common to call A a “family of sets” or a “class of sets” rather than call it a “set of sets”.

Literature

1. Zhunisbekova D.A., Ashirbaev Kh.A. Higher Mathematics. - Shymkent: Publishing house “Alem”, 2018. – 360 p.