FUNCTION'S STUDY

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"Function's study" is the most interesting theme of mathematical calculus. Let's read it. First, let's start with monotonicity.

Finding the interval of functions' monotonicity. The behavior of function is closely connected with the values of their derivatives, and, especially, with the sign of the first- and second-order derivatives.

Theorem 1. If the differentiable function $y=f(x)$ increases on an interval $(a;b)$ then its derivative is positive $f'(x)>0$.

Theorem 2. If the function $y=f(x)$ is continuous on the interval $[a;b]$ and differentiable on this interval and $f'(x)>0$, then the function $y=f(x)$ increases on this interval.

We can formulate the theorem for the decreasing function also:

1. If the function $y=f(x)$ has the derivative on the interval $(a;b)$ and is decreasing on it then its derivative is $f'(x)<0$ on this interval.

2. If $y=f(x)$ is continuous on the interval $(a;b)$ and differentiable on it, and $f'(x)<0$ then function $y=f(x)$ is decreasing on the interval.

The intervals of the increase and decrease of function. To find the interval of the decrease (increase) of function, we must:

1. Find the domain of existence.

2. Differentiate the function and equal the derivative to zero. Solving this equation, we find roots $x_1, x_2, \ldots, x_k$ and divide the domain of existence into the monotonicity intervals by these points.

3. Defining the signs of the derivative at any point of each interval, define where the function increases and decreases.

Finding extrema. Finding maximum and minimum values of the continuous function on the closed interval.

The function $y=f(x)$ at the point $x=x_1$ has the local maximum if the value at the point $x=x_1$ is larger than the rest of the values in a neighborhood of this point.

Another way: The function $y=f(x)$ reaches the local maximum as $x=x_1$ if

$$f(x_1+\Delta x)-f(x_1)<0,$$

at any Δx.

The function $y=f(x)$ has the local minimum at the point $x=x_2$ if

$$f(x_2+\Delta x)-f(x_2)>0,$$

at any Δx.

Definitions of *max* and *min* of functions are under a common title - the extrema of function; points *max* and *min* are called the points of extrema. We can find them applying the following.

Theorem. (The necessary condition of the extrema.) If the differentiable function $y=f(x)$ on $(a;b)$ has a maximum or a minimum at a point $x=x_1$ then its derivative at this point becomes zero.

The points, at which the derivative of function equals zero or is discontinuous, are called critical, controversial (there may be the extrema at them).

The analysis of function at critical points is based on the theorem (the sufficient conditions of existence of the extrema of function).

Suppose that $y=f(x)$ is continuous on an interval and is differentiable at all points of this interval containing the point, probably, except the point itself.

Theorem. If passing over the critical point $x=x_1$ the derivative of function changes its sign from + to –, then this point obtains the maximum. If passing over the critical point $x=x_1$, the derivative of function changes its sign from – to +, then this point obtains the minimum, i.e.,

(a) $\begin{cases} f'(x) > 0, x < x_1 \\ f'(x) < 0, x > x_1 \end{cases}$, at the point $x=x_1$ the function has its *max*.

(b) $\begin{cases} f'(x) < 0, x < x_1 \\ f'(x) > 0, x > x_1 \end{cases}$, at the point $x=x_1$ the function has its *min*.

However, we should remark that conditions (a) and (b) must hold true for all points (values of x) close enough to the point $x=x_1$.

Theorem. Suppose that $y=f(x)$ is continuous on the interval (a,b) at all interior points and has the following derivatives $f'(x_1)=0$, $f''(x_1)\neq 0$. Thus, if $f''(x_1)<0$ then the function has a maximum at this point; if $f''(x_1)>0$ the function has a minimum at this point.

Convexity, concavity, points of inflexion, asymptotes of functions' graph.

A curve $y=f(x)$ is called convex (concave) if all points of this curve is below (above) its tangents.

Theorem. Suppose that a function $y=f(x)$ has the second derivative at all points of the interval (a,b). If the second derivative on the interval is negative (positive), i.e., $f''(x)<0$ $(f''(x)>0)$ then the curve on this interval is convex (concave).

The sufficient conditions. Denote the curve by the equation $y=f(x)$. If y'' of the given function at the point $x=a$ becomes zero, or $y''(a)$ does not exist and, while passing over the point $x=a$, the second derivative y'' changes its sign then the point $x=a$ is an inflection.

Literature

1.Zhunisbekova D.A., Ashirbaev Kh.A. Higher Mathematics. - Shymkent: Publishing house "Alem", 2018. – 360 p.